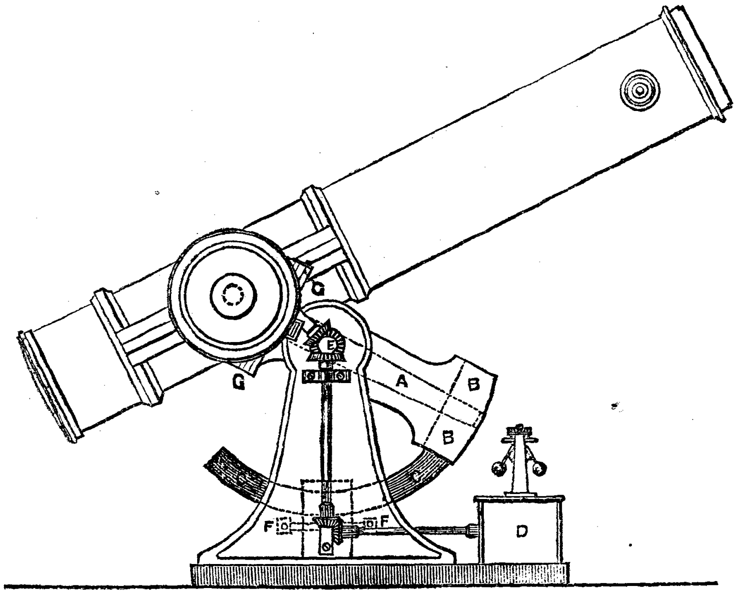
screw on the hour-circle G G, and thus drives the instrument; as the wheel E E runs loose on the spindle, and the distance between the wheel driven by E, and the main driving-screw on the hour-circle remains unaltered, it is evident that a change in the angle of position of the Polar axis does not interfere so as to prevent the instrument being freely driven by the clock.

As I have before stated, the suggestion that such an instrument should be made, was the Astronomer Royal's; and I have received efficient assistance from Mr. De La Rue in perfecting this contrivance.



On the Motion of Matter projected from the Sun: with special reference to the Outburst witnessed by Prof. Young of America. By Richard A. Proctor, B.A. (Cambridge.)

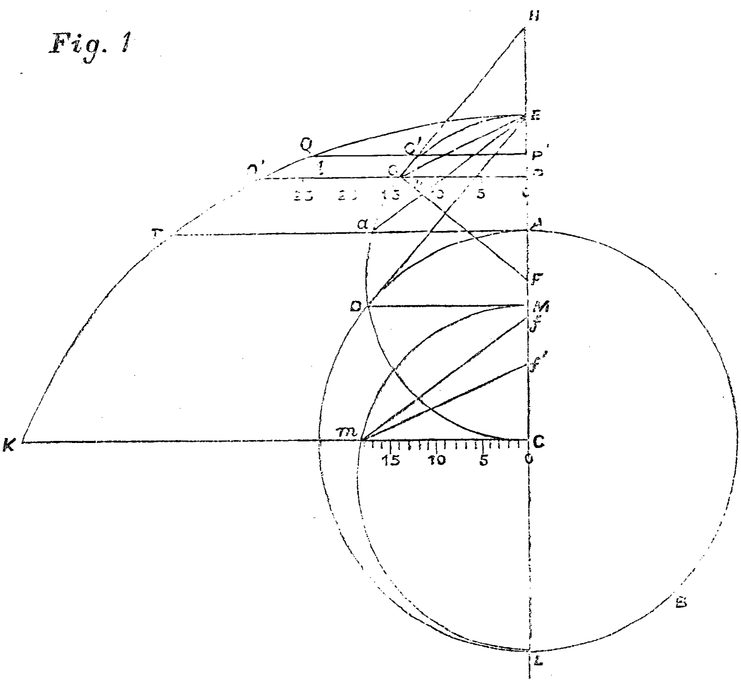
Whatever opinion we may form as to the way in which the matter of certain solar prominences is propelled from beneath the photosphere, there can be little question that such propulsion really takes place. It seems clear indeed that some prominences, more especially those seen in the Sun's polar and equatorial regions, are formed—or rather make their appearance—in the upper regions of the solar atmosphere, and even assume the appearance of eruption prominences by an extension *downwards*, somewhat as a waterspout simulates the appearance of an uprushing column of water though really formed by a descending movement. But it is certain that other prominences are really phenomena of eruption, because Respighi, Zöllner, and Young (probably also Lockyer and Secchi) have been able actually to watch the uprush of the flowing hydrogen.

In the case of any matter thus erupted, we shall clearly obtain

an inferior limit for the value of the initial velocity of outrush, if we assume that the apparent height reached by the matter is the real limit of its upward motion (that is, that there is no foreshortening) and that the solar atmosphere exercises no appreciable influence in retarding the motion. The latter supposition is, however, wholly untenable under the circumstances, while the former must in nearly all cases be erroneous; and I only make these suppositions in order to simplify the subject, noting that their effect is to reduce the estimated velocity of outrush to its lowest limiting value.

We are to deal then, for the present, with the case of matter flung vertically upwards from the Sun's surface and subject only to the influence of solar gravity; I propose to consider the time

Fig. 1



Illustrating the motion of a body descending from rest towards a globe attracting according to the law of nature.

of flight between certain observed levels, not the mere vertical distance attained by the erupted matter; and (as I wish to deal with cases where a great distance from the Sun has been attained) it will be necessary to take into account the different actions of solar gravity at different distances. Zöllner, in dealing with prominences of moderate height, has regarded the solar gravity as constant; but this is evidently not admissible when we come to deal with matter hurled to a height of 200,000 miles, since at that height solar gravity is reduced to less than one-half the value it has at the surface of the Sun.

It is easy to obtain the required formula; and though it is doubtless contained in all treatises on Dynamics, it will be as well to run through the work in this place. In reducing the formula

I have noticed a neat geometrical illustration (and partial proof) which I do not remember to have seen in any book. It not only presents in a striking manner the varying rate at which a body falls towards a centre attracting according to the law of Nature, but it supplies a means whereby the time of flight between any given distances may be readily obtained from a simple construction.

Let C be the centre of a globe A B D, of radius R, and attracting according to the law of nature; let gravity at the surface of the globe be represented by g. Then the attraction exerted at a unit of distance, if the whole mass of the globe were collected at a point, would be $g R^2$.

Let a particle falling from rest at E reach the point P in time t; and let $AE = H$, and $CP = x$. Then the equation of motion is

$$\frac{d^2 x}{d t^2} = -\frac{g R^2}{x^2},$$

giving

$$\left(\frac{d x}{d t}\right)^2 = v^2 = \frac{2 g R^2}{x} + C;$$

so that, since the particle starts from rest at a distance R + H from C, we have

$$0 = \frac{2 g R^2}{R+H} + C.$$

For convenience write D for R + H; then we have

$$\left(\frac{d x}{d t}\right)^2 = v^2 = 2 g R^2 \left(\frac{1}{x} - \frac{1}{D}\right)$$

$$= \frac{2 g R^2}{D}\left(\frac{D - x}{x}\right) \qquad (1)$$

Thus

$$R \sqrt{\frac{2 g}{D}} \cdot \frac{d}{d x} = \frac{x}{\sqrt{D x - x^2}}.$$

Integrating we have

$$R \sqrt{\frac{2 g}{D}} \cdot t = \sqrt{D x - x^2} - \frac{D}{2} \cos^{-2}\left(\frac{D - 2 x}{D}\right) + C$$

But $t = 0$, $x = D$; so that $C = \frac{D \pi}{2}$,

hence we have

$$R \sqrt{\frac{2 g}{D}}\, t = \sqrt{D x - x^2} + \frac{D}{2} \cos^{-1}\left(\frac{2 x - D}{D}\right) \quad (2)$$

(where D is equal to the radius of the globe added to the height from which the particle is let fall.)

Equation (1) gives the velocity acquired in falling (from rest) from a height H to a distance x from the centre, and (2) gives the time of falling to that distance. Now the geometrical illustration to which I have referred, relates to the deduction of (2) from (1). We see from (1) that at the point P

$$v^2 = \frac{2 g R^2}{D} \left(\frac{D - x}{x} \right).$$

Bisect C E in F, and describe the semicircle C D E; then if D E is a tangent to the circle D A B, and if D M is drawn perpendicular to C E,

$$\text{C M} = \frac{\text{C D}^2}{\text{C E}} = \frac{R^2}{D};$$

so that

$$v = \sqrt{2 g \,.\, \text{C M}} \sqrt{\frac{\text{P E}}{\text{C P}}}. \qquad (\alpha)$$

But if close by G, either on the tangent G H or on the arc G E, we take G′ and draw G′ P′ perpendicular to C E, and G′ n perpendicular to G P, we have

$$\frac{\text{G G}' + \text{G}n}{\text{P P}'} = \frac{\text{G F} + \text{F P}}{\text{G P}} = \frac{\text{C P}}{\sqrt{\text{C P} \,.\, \text{P E}}}$$

$$= \sqrt{\frac{\text{C P}}{\text{P E}}}.$$

Hence, from (α),

$$\frac{v}{\sqrt{2 g \,.\, \text{C M}}} = \frac{\text{P P}'}{\text{G G}' + \text{G}n},$$

so that

$$\left\{\begin{array}{c}\text{the vel.} \\ \text{at P}\end{array}\right\} : \left\{\begin{array}{c}\text{velocity acquired in falling through} \\ \text{space C M, under const. accel. force } g\end{array}\right\}$$

$$:: \left\{\begin{array}{c}\text{elem. space} \\ \text{P P}'\end{array}\right\} : \left\{\begin{array}{c}\text{sum of elementary} \\ \text{spaces G G}' \text{ and G} n.\end{array}\right\}$$

Therefore the falling particle traverses the space P P′, in the same time that a particle travelling with the velocity acquired in falling through space C M under constant accelerating force g, would traverse the space G G′ + G n. It follows obviously that the time in falling from E to P is the same as would be occupied by a particle in traversing the space (arc E G + G P), with the

velocity acquired in falling through the space C M under a constant accelerating force g. This amounts to saying that

$$t = \frac{\text{P G} + \text{arc G E}}{\sqrt{2\,g\,.\,\text{C M}}},$$

or that

$$\text{R}\sqrt{\frac{2\,g}{\text{D}}}\,.\,t = \sqrt{\text{P E}\,.\,\text{P C}} + \text{C F arc G E}$$

$$= \sqrt{(\text{D}-x)\,x} + \frac{\text{D}}{2}\cos^{-1}\left(\frac{2\,x-\text{D}}{\text{D}}\right)$$

as before.

The relation here considered affords a very convenient construction for determining times of descent in any given cases. For, if P G be produced to Q so that G Q = arc G E, Q lies on a semi-cycloid K Q C, having C E as diameter; and the relative time of flight from E to any point in A E is at once indicated by drawing through the point an ordinate parallel to C K. The actual time of flight in any given case can also be readily indicated. For let T be the time in which L C would be described with the velocity acquired in falling through a distance equal to L C under accelerating force g; and on L M describe the semicircle L m M; then clearly C m ($= \sqrt{\text{C L}\,.\,\text{C M}}$) will be the space described in time T with the velocity acquired in falling through the space C M under accelerating force g; and we have only to divide C m into parts corresponding to the known time-interval T, and to measure off distances equal to these parts on P Q to find the time of traversing P Q with this uniform velocity, *i.e.*, the time in which the particle falls from E to P. The division in the figure illustrates such measurements in the case of the Sun, the value of T being $18\frac{2}{3}$ minutes.

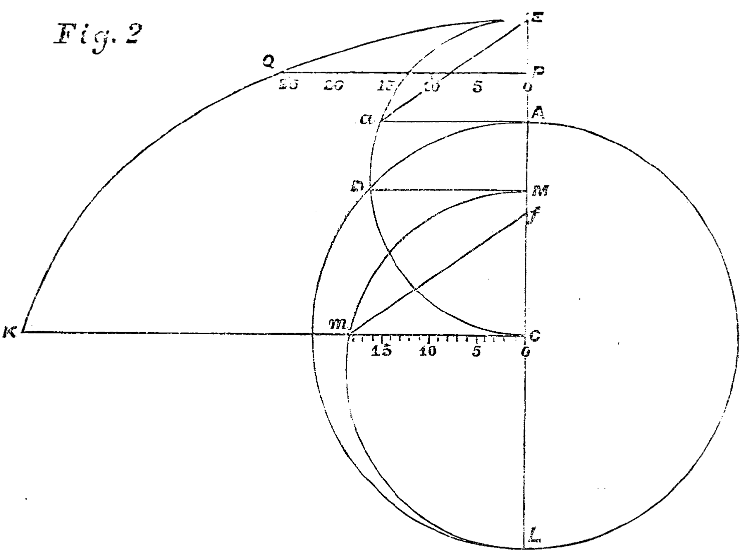
Moreover it is not necessary to construct a cycloid for each case. One carefully constructed cycloid * will serve for all cases, the radius C A being made the geometrical variable. As an instance of the method of construction, I will take Prof. Young's recent remarkable observation of a solar outburst, premising that I only give the constructions as illustrations, and that a proper calculation will follow.

Prof. Young saw wisps of hydrogen carried from a height of 100,000 miles to a height exceeding 200,000 miles from the Sun's surface in ten minutes. It is assumed here that there was no

* It is easy to construct a cycloid with great accuracy, nor need the methods available be here considered. But it is worthy of notice that if a wire coil be formed into a helix having the thread inclined 45° to the axis (which can be readily effected by the method which De Morgan suggested to Admiral Smyth,—see *Bedford Cycle*), and this thread placed on a horizontal table when the Sun is 45° high (the vertical plane including the Sun and the axis of the helix) the coil will throw a cycloidal shadow. (This arrangement is only presented in this way to show exactly how the point of projection should be placed with respect to the helix and plane; of course the projection can be obtained without the Sun's aid.)

foreshortening. The height, 100,000 miles, was determined by estimation; but the ultimate height reached by the hydrogen wisps (that is, the elevation at which they vanished as by a gradual dissolution) results from the mean of three carefully executed and closely accordant measures. This mean was $7' \, 49''$, corresponding to a height of 210,000 miles. We may safely take 100,000 miles as the vertical range actually traversed, and 200,000 miles as the extreme limit attained. We need not inquire whether the hydrogen wisps were themselves projected from the photosphere,—most probably they were not,—but if not, yet beyond question there was propelled from the Sun some matter which by its own motion caused the hydrogen to traverse the above-mentioned range in the time named. We shall be underrating the velocity of expulsion, in regarding this matter as something solid propelled through a non-resisting medium, and attaining an extreme range of 200,000 miles. What follows will show whether this supposition is admissible.

Fig. 2



Illustrating the construction for determining time of descent of a particle from rest towards a globe attracting according to the law of nature.

Now g for the Sun, with a mile as the unit of length and a second for the unit of time is $\cdot 169$, and R for the Sun is 425,000. Thus the velocity acquired in traversing R under uniform force g

$$= \sqrt{2 g \, . \, \mathrm{R}}$$

$$= \sqrt{338 \times 425}$$

$$= 379, \text{ very nearly}$$

(this is also the velocity acquired under the Sun's actual attraction by a body moving from an infinite distance to the Sun's surface.)

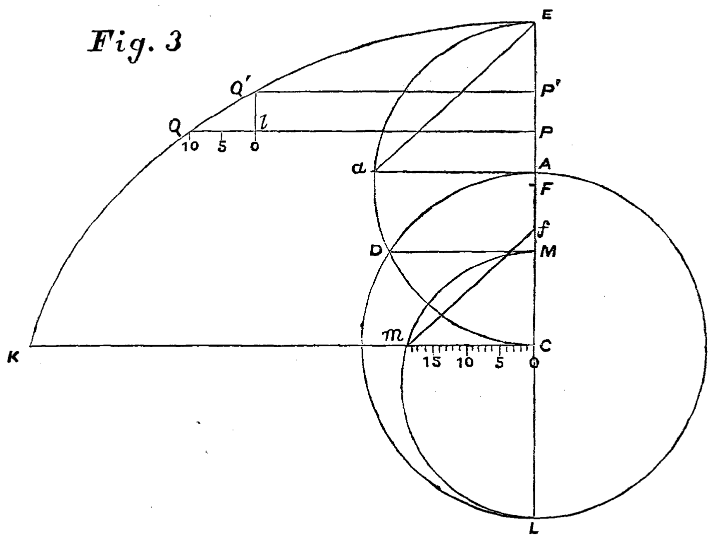
And a distance 425,000 would be traversed with this velocity in $18^m\ 40^s$ (= T.)

Let K Q E, *fig.* 2, be our semi-cycloid (available for many successive constructions if these be only pencilled), and C D E half the generating circle.

Then the following is our construction:—Divide E C into $6\frac{1}{4}$ equal portions, and let E P, P A be two of these parts, so that E A represents 200,000 miles and C A 425,000 miles (the Sun's radius). Describe the semicircle A D L about the centre C and draw D M perpendicular to E C; describe half circle M *m* L. Then *m* C represents T where the ordinate P Q represents time of falling from E to P.

$T = 18^h\ 50^m$, and P Q (carefully measured) is found to correspond to about twenty-six minutes.

Fig. 3



Illustrating the construction for determining time of descent between given levels, when a body descends from rest at a given height towards a globe attracting according to the law of nature.

Thus a body propelled upwards from A to E would traverse the distance P E in twenty-six minutes. But the hydrogen wisps watched by Prof. Young traversed the distance represented by P E in ten minutes. Hence either E was not the true limit of their upward motion, or they were retarded by the resistance of the solar atmosphere. Of course if their actual flight was to any extent foreshortened, we should only the more obviously be forced to adopt one or other of those conclusions.

But now let us suppose that the former is the correct solution; and let us inquire what change in the estimated limit of the uprush will give ten minutes as the time of moving (without resistance) from a height of 100,000 to a height of 200,000 miles. Here we shall feel the advantage of the constructive method; for to test the matter by calculation would be a long process, whereas each construction can be completed in a few minutes.

Let us try 375,000 miles as the vertical range. This gives C E = 800,000 miles, and our construction assumes the following appearance. We have A C to represent 425,000; A P = P P′= 100,000 miles; and Q l or (P Q —P′ Q′) to represent the time of flight from P to P′.

The semicircles A D L, M m L give us m C to represent T or $18^h\ 50^m$; and Q L carefully measured is found to correspond to rather less than ten minutes. It is, however, near enough for our purpose.

It appears, then, that if we set aside the probability, or rather the certainty, that the Sun's atmosphere exerts a retarding influence, we must infer that the matter projected from the Sun reached a height of 375,000 miles, or thereabouts. This implies an initial velocity of about 265 miles per second.*

But it will be well to make an exact calculation,—not that any very great nicety of calculation is really required, but—to illustrate the method to be employed in such cases, as well as to confirm the accuracy of the above constructions.

In equation (2) put $\sqrt{2\,g\,\mathrm{R}}=379$; R $=425,000$; D $= 625,000$; and $x = 525,000$; values corresponding to Prof. Young's observations. It thus becomes—

$$\sqrt{\frac{425}{625}}\,(379)\,t = \sqrt{(100,000)\,(525,000)} + 312,500\ \cos^{-1}\left(\frac{1050-625}{625}\right),$$

or

$$379\sqrt{17}\,.\,t = 250,000\sqrt{21} + 1,562,500\ \cos^{-1}\left(\frac{17}{25}\right),$$

$$1562{\cdot}7\ t = 1,145,100 + 1,285,800 = 2,430,900$$

$$t = 1556^s = 25^m\ 56^s.$$

This then is the time which would have been occupied in the flight of matter from a height of 100,000 to a height of 200,000 miles, if the latter height had been the limit of vertical propulsion in a non-resisting medium.

In order to deduce the time of flight t' between the same levels, for the case where the total vertical range is 375,000 miles, we have, putting t_1 for time of *fall* to 200,000 miles above Sun's surface, and t_2 for time of fall to 100,000 miles, the equation,

* This value is of course deduced directly from (1); but it is worthy of notice that it can be deduced at once from *fig.* 3, by drawing A a parallel to K C, and $m f$ parallel to a E; then C f represents the required velocity, C L representing 379 miles per second. A similar construction will give the velocity at P, P′ &c. Applied to *fig.* 1, it gives C f to represent the velocity at A, C f' to represent the velocity at P; $m f$ and $m f'$ being parallel to a E and G E respectively. Applied to the case dealt with in *fig.* 2, we get C f to represent the velocity at A, where E is the limit of flight; C f is found to be rather more than $\frac{5}{9}$ of C L; so that the velocity at A is rather more than 210 miles per second.

$$\sqrt{\frac{425}{800}}\,(379)\,t_1 = \sqrt{(175{,}000)\,(625{,}000)} + (400{,}000)\cos^{-1}\left(\frac{125-80}{80}\right),$$

and

$$\sqrt{\frac{425}{800}}\,(379)\,t_2 = \sqrt{(275{,}000)\,(525{,}000)} + (400{,}000)\cos^{-1}\left(\frac{105-80}{80}\right),$$

giving

(since $t_2 - t_1 = t'$)

$$\sqrt{\frac{425}{800}}\,(379)\,t' = 25{,}000\left\{\sqrt{11\times 21} - \sqrt{7\times 25}\right\} + 400{,}000\left\{\cos^{-1}\left(\frac{5}{16}\right) - \cos^{-1}\left(\frac{9}{16}\right)\right\}$$

$$276\cdot 25\, t' = 49{,}250 + 111{,}816 = 161{,}066,$$

$$t' = 583^{s} = 9^{m}\,43^{s}.$$

This is very near to Prof. Young's ten minutes. I had found that an extreme height of 400,000 miles gave $9^{m}\,24^{s}$ for the time of flight between vertical altitudes 100,000 miles and 200,000 miles. It will be found that a height of 360,000 miles gives $9^{m}\cdot 585$, which is sufficiently near to Prof. Young's time.

Now to attain a height of 360,000 miles a projectile from the Sun's surface must have an initial velocity

$$= \sqrt{2gR}\;.\;\sqrt{\frac{360{,}000}{785{,}000}} = 379\sqrt{\frac{72}{157}}$$

$$= 257 \text{ miles per second.}$$

The eruptive velocity, then, at the Sun's surface, cannot possibly have been less than this. When we consider, however, that the observed uprushing matter was vapourous, and not very greatly compressed (for otherwise the spectrum of the hydrogen would have been continuous and the spectroscope would have given no indications of the phenomenon), we cannot but believe that the resisting action of the solar atmosphere must have enormously reduced the velocity of uprush before a height of 100,000 miles was attained, as well as during the observed motion to the height of 200,000 miles. It would be safer indeed to assume that the initial velocity was a considerable multiple of the above-mentioned velocity, than only in excess of it by some moderate proportion. Those who are acquainted with the action of our own

atmosphere on the flight of cannon-balls (whereby the range becomes a mere fraction of that due to the velocity of propulsion), will be ready to admit that hydrogen rushing through 100,000 miles even of a rare atmosphere, with a velocity so great as to leave a residue sufficient to carry the hydrogen 100,000 miles in the next ten minutes, must have been propelled from the Sun's surface with a velocity many times exceeding 257 miles per second, the result calculated for an unresisted projectile. Nor need we wonder that the spectroscope supplies no evidence of such velocities, since when motions so rapid existed, others of all degrees of rapidity down to such comparatively moderate velocities as twenty or thirty miles per second would also exist, and the spectral lines of the hydrogen so moving would be too greatly widened to be discerned.

Now the point to be specially noticed is, that supposing matter more condensed than the upflung hydrogen to be propelled from the Sun during these eruptions, such matter would retain a much larger proportion of the velocity originally imparted. Setting the velocity of outrush, in the case we have been considering, at only twice the amount deduced on the hypothesis of no resistance (and it is incredible that the proportion can be so small), we have a velocity of projection of more than 500 miles per second; and if the more condensed erupted matter retained but that portion of its velocity corresponding to three-fourths of this initial velocity (which may fairly be admitted when we are supposing the hydrogen to retain the portion corresponding to so much as half of the initial velocity), then such more condensed erupted matter would pass away from the Sun's rule never to return.

The question may suggest itself, however, whether the eruption witnessed by Prof. Young might not have been a wholly exceptional phenomenon, and so the inference respecting the possible extrusion of matter from the Sun's globe be admissible only as relating to occasions few and far between. Now on this point I would remark, in the first place, that an eruption very much less noteworthy would fairly authorise the inference that matter had been ejected from the Sun. I can scarcely conceive that the eruptions witnessed quite frequently by Respighi, Secchi, and Young—such eruptions as suffice to carry hydrogen 80,000 or 100,000 miles from the Sun's surface—can be accounted for without admitting a velocity of outrush exceeding considerably the 379 miles per second necessary for the actual *rejection* of matter from the Sun. But apart from this it should be remembered that we see only those prominences which happen to lie round the rim of the Sun's visible disk, and that thus many mighty eruptions must escape our notice even though we could keep a continual watch upon the whole circle of the sierra and prominences (which unfortunately is very far from being the case).

It is worthy of notice that the great outrush witnessed by Prof. Young was not accompanied by any marked signs of mag-

netic disturbance. Five hours later, however, a magnetic storm began suddenly, which lasted for more than a day; and on the evening of September 7 there was a display of the aurora borealis. Whether the occurrence of these signs of magnetic disturbance were associated with the appearance (on the visible half of the Sun) of the great spot which was approaching or crossing the eastern limb at the time of Young's observation, cannot at present be determined.

I would remark, however, that so far as is yet known the disturbance of terrestrial magnetism by solar influences would appear to depend on the condition of the photosphere, and therefore to be only associated with the occurrence of great eruptions in so far as these affect the condition of the photosphere. In this case an eruption occurring close by the limb could not be expected to exercise any great influence on the Earth's magnetism; and if the scene of the eruption were beyond the limb, however slightly, we could not expect any magnetic disturbance at all, though the phenomena of eruption might be extremely magnificent.

In this connection I venture to quote from a letter addressed to me by Sir J. Herschel in March 1871 (a few weeks only before his lamented decease). The letter bears throughout on the subject of this paper, and therefore I quote more than relates to the association between terrestrial magnetism and disturbances of the solar photosphere.

After referring to Mr. Brothers' photograph of the corona (and remarking by the way that "the corona is certainly *extra*-atmospheric and *ultra*-lunar"), Sir John Herschel proceeds thus:—

"I can very well conceive great outbursts of vaporous matter from below the photosphere, and can admit at least the possibility of such vapour being tossed up to very great heights; but I am hardly yet exalted to such a point as to conceive a positive ejection of erupted particles with a velocity of two or three hundred miles per second. But now the great question of all arises: what *is* the *photosphere?* what are those intensely radiant *things*—scales, flakes, or whatever else they be—which really *do* give out all (or at least $\frac{99}{100}$ths of) the total light and heat of the Sun? and if the prominences &c. be eruptive, why does not the eruptive force scatter upwards and outwards this luminous matter? .... Through the kindness of the Kew observers I have had heliographs of the two great outbursting spots which I think I mentioned to you as having been non-existent on the 9th, and in full development on the 10th—both [being] large and conspicuous, and including an area of disturbance at least 2′ (54,000 miles) across. They were both nearly absorbed, or in rapid process of absorption, on the 11th. In my own mind I had set it down as pretty certain that the outbreak must have taken place *very* suddenly at somewhere about the intervening midnight. Well, now! The magnet's declination curves at Kew have been sent me, and, lo! while they had been going on as

smoothly as possible on the 6th, 7th, 8th, and 9th, and up to $11\frac{1}{2}$ P.M. on the latter day (9th), suddenly a great downward jerk in the curve, forming a gap as far as $3\frac{1}{2}$ A.M. on the 10th. Then comparative tranquillity till 11 A.M., and then (corresponding to the re-absorption of the spots) a furious and convulsive state of disturbance extending over the 11th and the greater part of the 12th. I wonder whether anything was shot out of those holes on that occasion! and, if so, what is going on in the inside of the Sun?"

Note on a special point in the determination of the Elements of the Moon's Orbit from Meridional Observations of the Moon. By George Biddell Airy, Astronomer Royal.

The magnitude of the Moon's apparent disk produces a difficulty in the reduction of observations of the Moon which does not occur in the case of the Sun or any of the Planets. In general, the two opposite limbs of the Moon (either those observed in Right Ascension or those observed in North Polar Distance) cannot be fully illuminated, and, in the great majority of instances, they cannot even be in a state approximating to that of full illumination; no attempt can be made to observe the centre; and therefore the only way of determining the apparent place of the centre is, to observe the fully illuminated limb, and to apply to the result a numerical correction representing the computed value of apparent semidiameter at the time of observation. This apparent semidiameter must be computed from a mean semidiameter inferred from observations of both limbs, when both limbs are so near to the state of full illumination, that the small correction depending on the deficiency of illumination of one limb can be calculated with great accuracy. Very few observatories furnish observations adapted to this purpose; and in consequence there will always be a little uncertainty about the value of the apparent semidiameter.

In correcting the observed Right Ascension of a limb, there will therefore always be a small uncertainty on the inferred Right Ascension of the centre, which has one sign before Full Moon and the opposite sign after Full Moon. It is well known that this introduces uncertainty with regard to the coefficients of the two important equations designated the "Variation" and the "Parallactic Equation." These effects, however, are sufficiently well known, and I shall not delay further on them.

The point to which I invite the attention of the Society is one which I believe has never been noticed, namely, the effect of the uncertainty of correction for semidiameter on the inferred results for North Polar Distance of the Moon's centre. I believe that it is not generally known to the investigators of Lunar Theory and